PH421 - Thermal and Statistical Physics

Assignment 11 - Apr 4, 2008

1. Maxwell's distribution of speeds - 1

Consider an ideal gas in equilibrium at temperature T, and composed of molecules of mass m. Show that:

$$< v >= \sqrt{\frac{8kT}{\pi m}} \tag{1}$$

$$v_{max} = \sqrt{\frac{2kT}{m}} \tag{2}$$

$$< v^2 >= \frac{3kT}{m} \tag{3}$$

where v_{max} is the most probable speed.

2. Maxwell's distribution of speeds - 2

(a) Show that the distribution of kinetic energies ϵ in an ideal gas that obeys the Maxwellian distribution of speeds is:

$$F(\epsilon)d\epsilon = 2\pi \left(\frac{1}{\pi kT}\right)^{3/2} \epsilon^{1/2} e^{-\frac{\epsilon}{kT}} d\epsilon \tag{4}$$

(b) Calculate the most probable kinetic energy in a Maxwellian gas.

3. Application of the equipartition theorem

Consider a diatomic molecule in which two atoms of mass M_1 and M_2 interact via an attractive potential. Assume that:

- The attractive potential can be approximated as a square term of the scalar distance r between the two atoms;
- The molecule moves freely in 3-D space;
- The atoms' motion relative to the center of mass can be described by three variables r, θ and ϕ. Recall that the corresponding terms in the Hamiltonian are square terms in these variables.

Find the specific heat capacity at constant volume C_V for a diffuse gas composed of these molecules.