PH421 - Thermal and Statistical Physics

Assignment 1 - Jan 7, 2008

Permutations, combinations and number of microstates.

Consider N balls, of which r are red and $(N-r)$ are blue. One is interested in knowing the number of *combinations* C_r^N of r red balls out of N balls.

(a) First, calculate the number of *permutations* of r balls out ot N balls, i.e., the way in which r balls can be ordered in N available "boxes".

(b) Then, realize that the order in which the r balls are located doesn't matter; this corresponds to finding the number of ways in which the r balls occupy their r places, which is equivalent to the permutation of r balls among themselves (or out of r balls).

(c) Divide the number found in (a) by that found in (b), to find that

$$C_r^N = \binom{N}{r} = \frac{N!}{r!(N-r)!} \tag{1}$$

(d) Work out a numerical example in which N=4, and $r = 2$. Draw all possible C_r^N combinations.

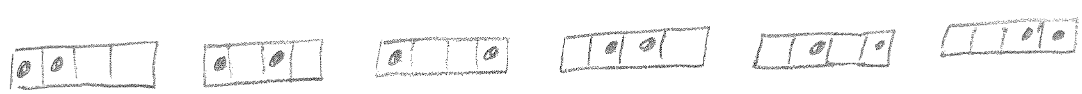
This simple problem can be interpreted in terms of a macrostate labeled by the number r, and to which there are associated $\Omega(r) = C_r^N$ microstates.

(a) : $N \cdot (N-1) \cdots \cdot (N-r+1)$

(b) $r!$

(c) $$C_r^N = \frac{N(N-1)(N-r+1)\cdot(N-r)\cdots\cdot 1}{r! \quad (N-r)\cdots\cdot 1} = \frac{N!}{r!(N-r)!} = \binom{N}{r}$$

(d) $$C_2^4 = \frac{4!}{2!\,2!} = \frac{4\times3\times2}{2\times2} = 6$$



Notice that, when using the number of combinations, we are tacitly assuming that one cannot label each ball. We shall return to this concept later.