PH421 - Thermal and Statistical Physics

Assignment 7 - Feb 20, 2008

1. Magnetization of a paramagnetic solid.

A paramagnetic solid with N spin 1/2 atoms is in thermal equilibrium at temperature T. Each atom has a magnetic dipole moment $\boldsymbol{\mu}$

(a) Using the definition of magnetization $\mathcal{M}$ and magnetic susceptibility χ,

$$\mathcal{M} = N \cdot \overline{\mu} \tag{1}$$

$$\chi = \frac{\mathcal{M}/V}{B/\mu_0} \tag{2}$$

to show that in the low-temperature/high-field limit,

$$\mathcal{M} \to N \cdot \mu. \tag{3}$$

(b) The *magnetization* $\mathcal{I}$ is defined as the density of the induced magnetization $\mathcal{M}$,

$$\mathcal{I} = \frac{\mathcal{M}}{V} \tag{4}$$

Consider a paramagnetic solid in which each atom has $\mu = 10^{-26}$ A m^2, immersed in a magnetic field $B = 1$ T. Find the density $n = N/V$ of magnetic dipoles needed to make $\mathcal{I} = B/\mu_0$, in the low-temperature approximation. Assuming that each atom is Fe, find the density ρ of the material.

2. Entropy and negative temperature of a paramagnetic solid

Consider a paramagnetic solid composed of N magnetic dipoles of magnetic moment μ. Call n the number of dipoles that are aligned along $\boldsymbol{B}$. Show that, for $n < N/2$, the magnetic temperature given by

$$\frac{1}{T} = \frac{\partial S}{\partial E} \tag{5}$$

is negative, and it is positive otherwise.

3. Problem 3.1 from textbook

4. Problem 3.4 from textbook

5. *(For extra credit only)*

Consider the heat capacity at constant B for a paramagnetic solid of N atoms of magnetic moment μ,

$$C_B = kx^2 sech^2(x) \tag{6}$$

Show that the following equation applies:

$$\Delta E = \int_0^\infty C_B dT = \mu B N \tag{7}$$