PH673 - High Energy Astrophysics

Assignment 19 - Nov. 14, 2007

1. (a) Prove the Lorentz trasformation for the three-acceleration $\boldsymbol{a}$ ($\boldsymbol{a} = d^2\boldsymbol{x}/dt^2$)

$$a_x = \frac{a'_x}{\gamma^3\sigma^3} \tag{1}$$

$$a_y = \frac{a'_y}{\gamma^2\sigma^2} - \frac{u'_y \cdot v}{c^2}\frac{a'_x}{\gamma^2\sigma^3} \tag{2}$$

$$a_z = \frac{a'_z}{\gamma^2\sigma^2} - \frac{u'_z \cdot v}{c^2}\frac{a'_x}{\gamma^2\sigma^3} \tag{3}$$

where $\sigma \equiv 1 + v \cdot u'_x/c^2$, and v is the boost in the x direction.

(b) Find simplified transformations involving $a_\parallel$ and $a_\perp$ in the case in which Σ' is the instantaneous rest frame of the particle.

2. Consider a source of size R that emits synchrotron radiation, in the presence of a magnetic field $\boldsymbol{B}$=1 μG; the emitting electrons are distributed as a power-law of index $p = 2$ ($N(E)dE \propto E^{-2}dE$), where $E_{min} = 10$ MeV and $E_{max} = 10$ GeV.

(a) Find the approximate maximum frequency of the synchrotron radiation;

(b) Find the spectral index of the synchrotron intensity in the low frequency limit;

(c) Find the spectral index of the radiation at high frequency, where the source is optically thin, and draw an approximate graph of the behavior of I_ν at all frequencies.