

PH689-01 – Fall 2006
Statistical methods for physics and astrophysics
SOLUTIONS to Assignment #6

1. Problem 3.2 from Bevington, 2nd edition.

$$A = \frac{\pi}{4} D^2$$

Since the function is a power, use (3.28) with $a = \frac{\pi}{4}$, $b=2$

$$\frac{\sigma_A}{A} = b \cdot \frac{\sigma_D}{D} = 2\%.$$

Same conclusions if you use the radius, since $r = \frac{1}{2}D$ and $\frac{1}{2}$ is a (deterministic) constant.

2. Problem 3.3 from Bevington, 2nd edition.

The resistance is given by

$$R = a \frac{L}{\pi r^2}$$

$$\sigma_R^2 = \sigma_L^2 \left(\frac{dR}{dL}\right)^2 + \sigma_r^2 \left(\frac{dR}{dr}\right)^2$$

assuming that L and r are measured independently.

$$\frac{dR}{dL} = \frac{R}{L}$$

$$\frac{dR}{dr} = -\frac{2R}{r}$$

$$\Rightarrow \left(\frac{\sigma_R}{R}\right)^2 = \left(\frac{\sigma_L}{L}\right)^2 + 4\left(\frac{\sigma_r}{r}\right)^2$$

In order for the relative uncertainty $\left(\frac{\sigma_R}{R}\right)$ to be equally coming from r and L , then the relative uncertainty in r should be twice as small as that in L .

3. Problem 3.5 from Bevington, 3rd edition.

Snell's law is $n_2 \sin(\theta_2) = n_1 \sin(\theta_1)$, n_1 is known.

From the measurements of θ_1 and θ_2 one obtains n_2 as

$$n_2 = n_1 \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

$$\frac{dn_2}{d\theta_1} = \frac{n_1 \cos(\theta_1)}{\sin(\theta_2)} \quad \frac{dn_2}{d\theta_2} = -\frac{n_1 \sin(\theta_1) \cos(\theta_2)}{\sin(\theta_2)^2}$$

$$\sigma_{n_2}^2 = n_2^2 (\sigma_{\theta_1}^2 \cdot \cot(\theta_1)^2 + \sigma_{\theta_2}^2 \cdot \cot(\theta_2)^2)$$

$n_2 = 1.503$ from the best-fit values measured.

$$\sigma_{n_2}^2 = 5.8 \cdot 10^{-4} \text{ (remember to use radians)} \Rightarrow n_2 = 1.503 \pm 0.024$$