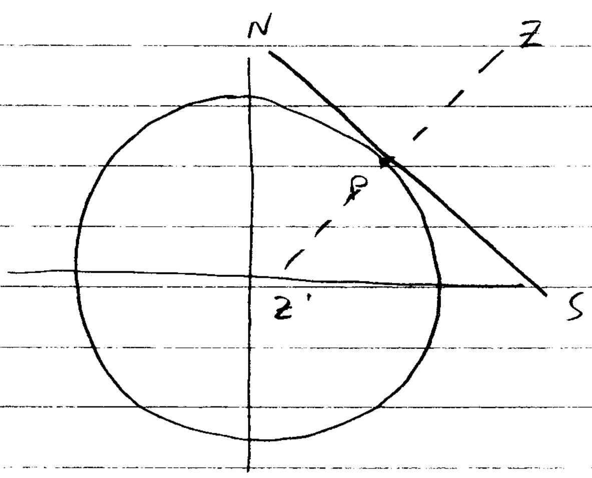
COORDINATE SYSTEMS

In order to locate a celestial source two coordinates are needed, since the sources appear projected onto the "celestial sphere"

<u>Altitude - Azimuth coordinate system</u> (Horizontal coordinates)

we need to develop first a few definitions:

<u>Local Horizon</u>: it is a plane tangent to the local position of the observer on the spherical Earth

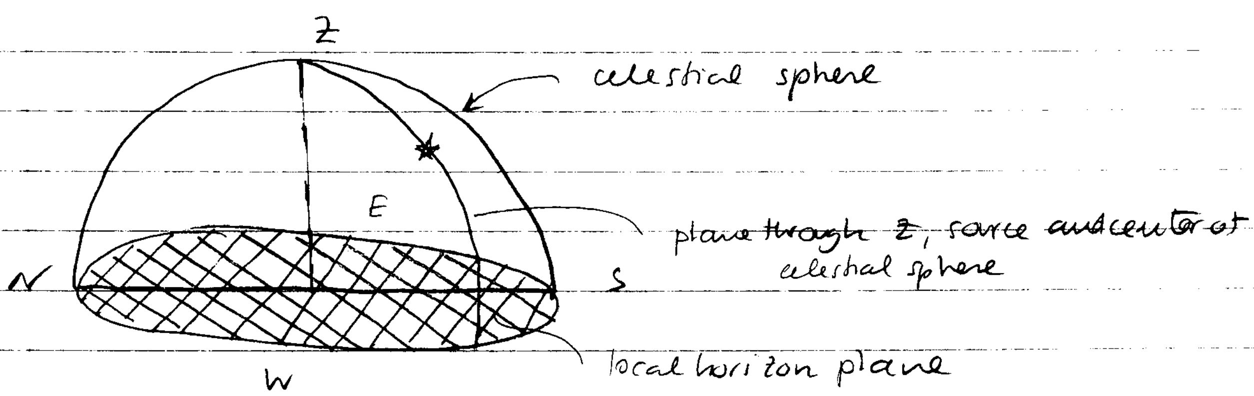


NS : local horizon at location P

Z : zenith

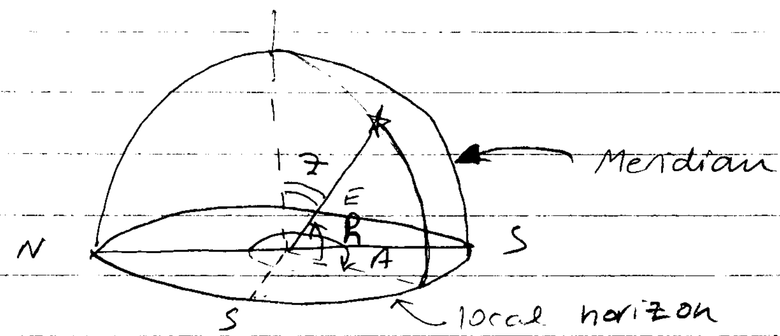
Z' : nadir

<u>Great circle</u> : is the curve resulting from the intersection of a sphere with a plane passing through the center of that sphere.



Consider a source on the celestial sphere, and the great circle given by the intersection of the celestial sphere with the plane that goes through the center of the sphere, the zenith and the source

• The altitude h is defined as the angle from the horizon to the source, along such plane



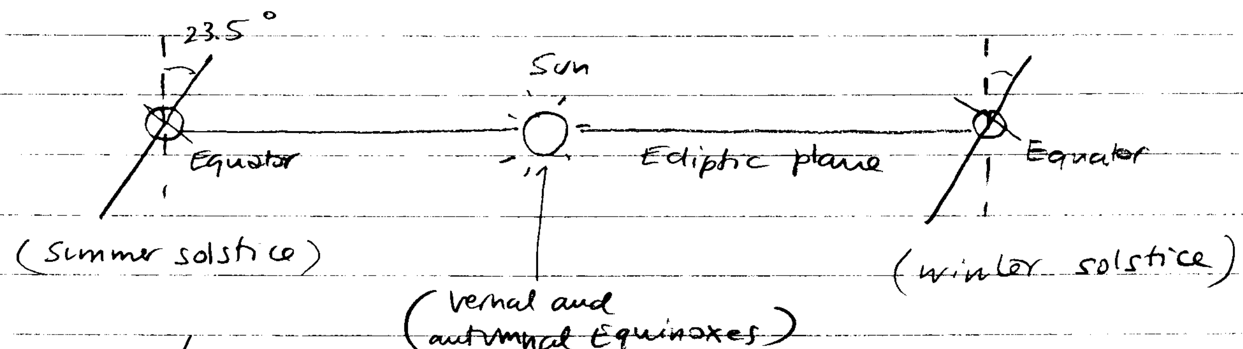
• The zenithal distance z is simply $z = 90° - h$

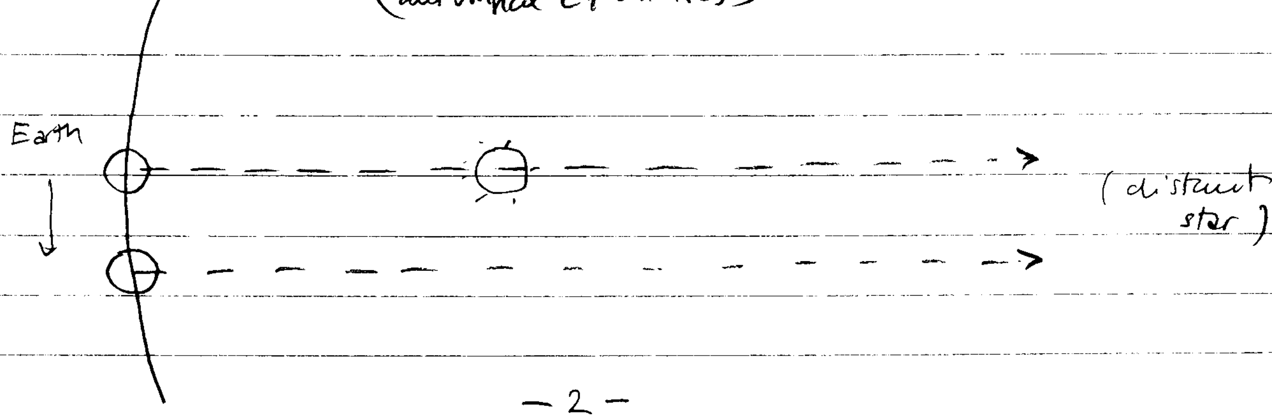
• The azimuth A is defined as the angle along the horizon, measured from the North due East.

• The meridian is another great circle given by the intersection of the celestial sphere with a plane going through the zenith and intersecting the local horizon at North and South.

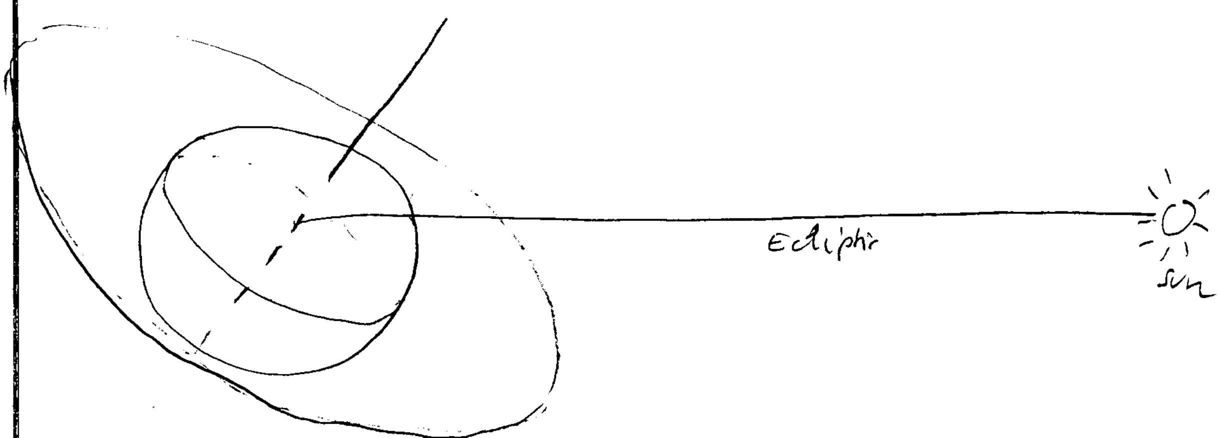
The daily and yearly motion of Earth

The Earth orbits around the Sun along a plane called ecliptic, which has an inclination with respect to the terrestrial equator of $\sim 23.5°$



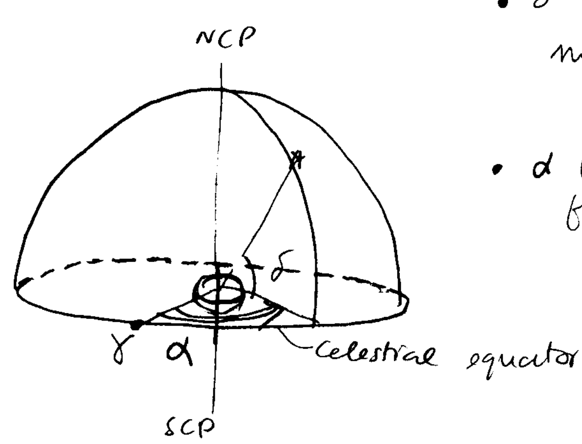


The *celestial equator* is defined by a plane through the Earth's equator, and extending out to the celestial sphere



The *equatorial coordinate system*

Consider the celestial equator in guise of the local horizon, and define the following coordinates:

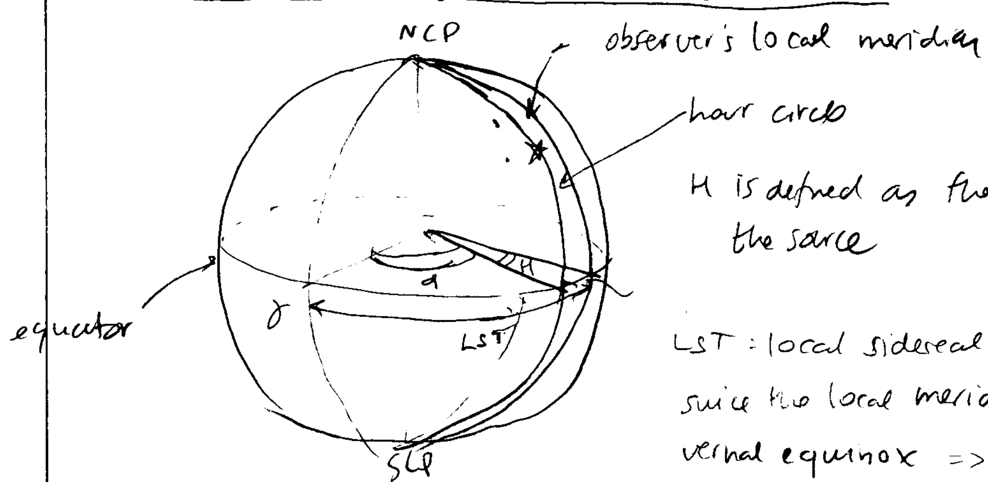


- δ (declination) is measured in degrees north or south of the celestial equator
- α (right ascension) is measured along the celestial equator from the position of the vernal equinox

The circle through the object's position and the North and South celestial pole is called the *hour circle* of the object.
The right ascension can be measured in degrees or in hours, minutes and seconds, e.g.,

$$R.A = 12^h\ 14'\ 37'' \qquad (24^h = 360^\circ)$$

The equatorial coordinates do not participate in the Earth's rotation, therefore the observer's local meridian and the hour circle coincide every 24 hours.

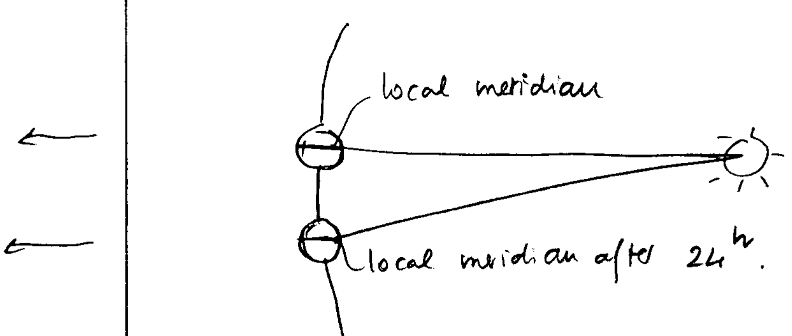


H is defined as the hour angle of the source

LST : local sidereal time, time elapsed since the local meridian passed the vernal equinox $\Rightarrow LST = H + \alpha$

Measures of time

The Earth completes one period of rotation around the Sun in approximately 365.26 days, therefore every day the Earth moves approximately 1° along the ecliptic



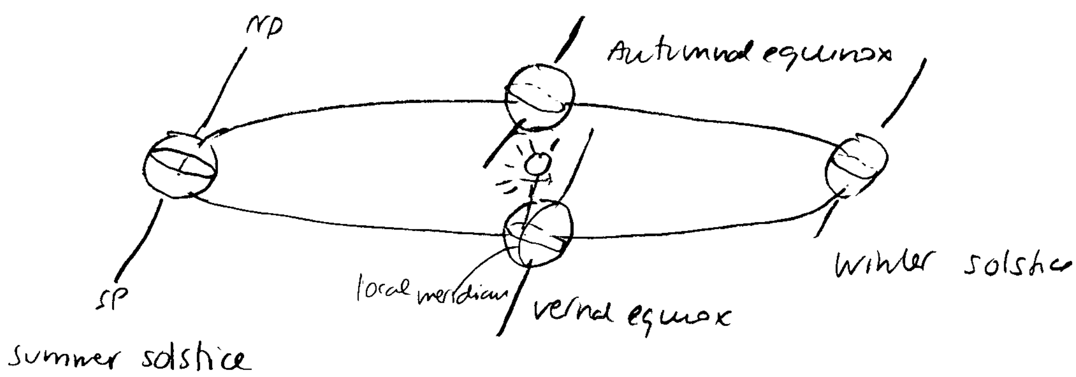
The Earth must rotate 360° to bring a given source to its meridian, but 361° to bring the Sun to its meridian
A period of 24 hours is known as a solar day, and it corresponds to the time between two consecutive meridian crossings of the Sun.
A sidereal day is defined as time between meridian crossings of distant sources, which required a 360° rotation only

Therefore, a sidereal day is

$$24^h \times \frac{360}{361} = 23.9335^h \simeq 23^h\,56^m$$

and the "fixed stars" and other distant celestial objects rise approximately 4 minutes earlier every day.

Consider the revolution of the Earth around the Sun:



The vernal and autumnal equinoxes are defined by the intersection between the ecliptic and the celestial equator

Throughout the day, the observer's local meridian changes its position on the celestial sphere because of the rotation of the Earth, and therefore the hour angle of a given fixed source also changes.

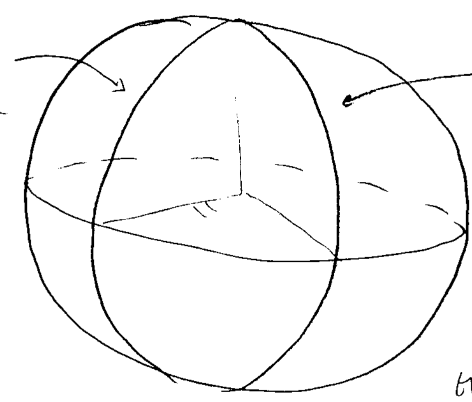
The local sidereal time (LST) can be defined via

$$LST = H + \alpha \qquad (1)$$

When $LST = \alpha$, this means that $H = 0$, or that the object is at its maximum elevation, above the local meridian. This is also called the transit of a source.

The local sidereal time can therefore be defined as "the time elapsed since the meridian traversed the vernal equinox," which is the point of reference for measuring the right ascension.

The coordinated universal time (UTC) is equivalent to the Greenwich Mean Time (GMT), which is the mean solar time at the reference meridian in Greenwich



GST is Greenwich sidereal time, different from the GMT (which is solar)

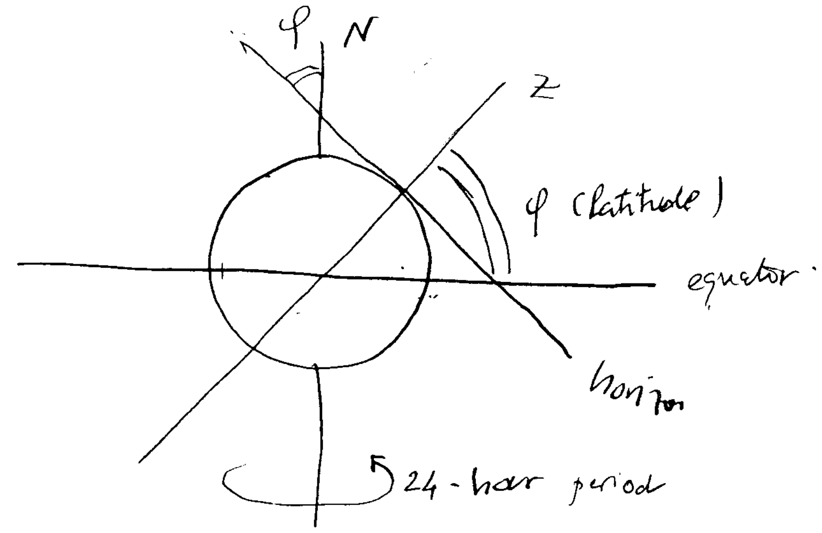
The LST corresponding to a given GST is a function of the observer's longitude; for example, an observer in Huntsville will be part of the CST (central Standard time) and therefore

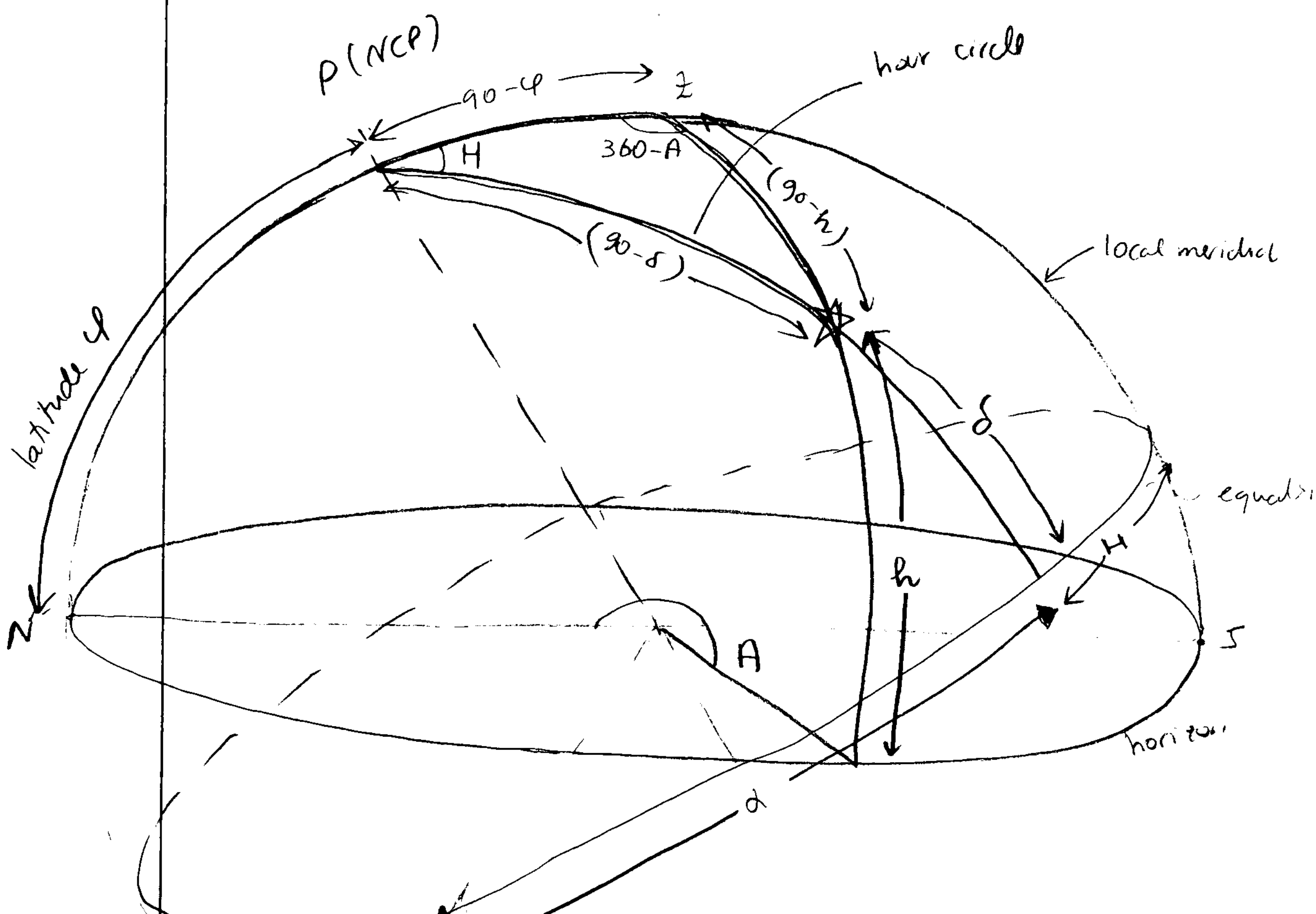
$$LST \simeq GST - 7 \text{ hours}. \quad (2)$$

Ex. Consider a source of $\alpha = 19^h\ 50^m\ 47^s$; the source will transit the local meridian at

$$LST = \alpha$$

Therefore the LST is equal to the right ascension α that passes through the local celestial meridian at that moment. In order to know at what local (solar) time this happens, one must know the GST to GMT conversion, which is rather complicated.

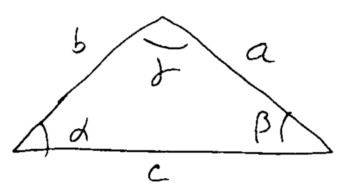




- The equator rotates with a cycle of $23^h 56'$, such that the hour circle overlaps the local meridian once per cycle

Conversion between horizontal and equatorial coordinates

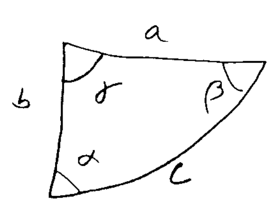
Recall the sine and cosine rule for a triangle:



Cosine rule $c^2 = a^2 + b^2 - 2a \cdot b \cos\gamma$

Sine rule: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

In a spherical triangle, similar relationships apply.



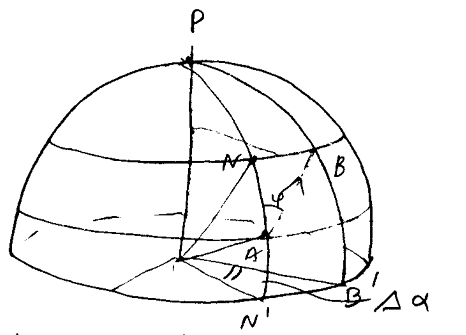
spherical cosine rule:

$$\begin{cases} \cos a = \cos b \cos c + \sin b \sin c \cos\alpha \\ \cos\alpha = -\cos\beta\cos\gamma + \sin\beta\sin\gamma\cos a \end{cases}$$

spherical sine rule:

$$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c}$$

Distance travelled and relationship to RA., dec.



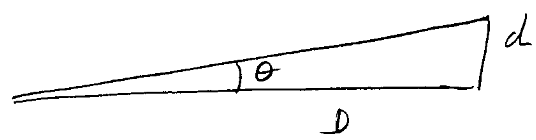
Consider two points of coordinates:

$$\begin{cases} A(\alpha, \delta) \\ B(\alpha+\Delta\alpha, \delta+\Delta\delta) \end{cases}$$

Consider the case in which the motion is "small", so that the small-angle approximation applies, and the geometry can be approximated to a plane instead of a sphere

$$d^2 = \overline{AB}^2 \simeq \overline{AN}^2 + \overline{BN}^2 \qquad (3)$$

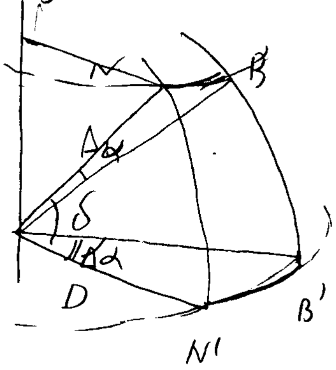
Consider the distance D from the center of the sphere, and the angle θ subtended by the distance d



$$\begin{cases} AB = d = D \cdot \sin\theta \simeq D \cdot \theta \\ AN = D \cdot \sin\Delta\delta \simeq D \cdot \Delta\delta \\ NB = (D \cdot \cos\delta) \cdot \sin\Delta\alpha \simeq D \cdot \Delta\alpha \cdot \cos\delta \end{cases}$$

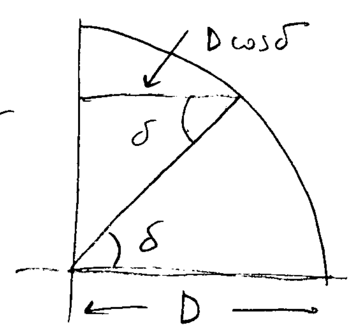
(small-angle approximation)

where $\Delta\alpha \cdot \cos\delta$ takes into account the "shrinking" of distances at high declination. In fact, NB and N'B' are both arcs of circles with smaller radii



$$\Delta\alpha = \frac{N'B'}{D} = \frac{NB}{D\cos\delta}$$

$$\Rightarrow NB = D \cdot \Delta\alpha \cdot \cos\delta$$



Therefore, the relationship between the angular distance θ subtended by the displacement d, and the change in the coordinates R.A and δ is

$$D^2 \theta^2 = D^2 \Delta\delta^2 + D^2 \Delta\alpha^2 \cos\delta^2$$

$$\Rightarrow \quad \theta^2 = \Delta\delta^2 + \Delta\alpha^2 \cdot \cos^2\delta \qquad (4)$$