3. A rocket is launched from the surface of a planet with unknown mass M, and radius $r = 6.67 \cdot 10^5$ [m]. It is known that the planet is orbited by a small moon, which orbits the planet at a distance of $R = 6.67 \cdot 10^7$ [m] from the center of the planet with an orbital speed of $v = 100$ m/s. The rocket is propelled by rocket boosters, which provide a total work of $W = 5 \cdot 10^7$ [J] during the trajectory from the surface (distance r) to a distance of $2 \cdot r$.

(a) What is the mass of the planet?

Use the fact that gravity provides the centripetal acceleration,

$\frac{v^2}{R} = G\frac{M}{R^2} \Rightarrow M = \frac{v^2 \cdot R}{G} = 10^4 \cdot 6.67 \cdot 10^7/(6.67 \cdot 10^{-11}) = 10^{22}$

(b) Assume that the rocket (mass m=100 [kg]) starts with $v = 0$. What is the speed of the rocket at a distance of $2 \cdot r$?

Use conservation of energy:

$W = \frac{1}{2}m \cdot v^2 + (-\frac{G \cdot M \cdot m}{2 \cdot r} + \frac{G \cdot M \cdot m}{\cdot r}) = \frac{1}{2}m \cdot v^2 + \frac{G \cdot M \cdot m}{2 \cdot r}$

$\Rightarrow v^2 = \frac{W \cdot 2}{m} - \frac{G \cdot M}{r} = 10^8/10^2 - 6.67 \cdot 10^{-11} \cdot 10^{22}/(6.67 \cdot 10^5) = 10^6 - 10^6 = 0$

(c) (Extra points) Will the rocket excape the gravitational pull of the planet, and why?

No, since at $2 \cdot r$ its velocity is zero. In order to escape it will need to have a speed larger than the escape speed at that radius.

4. Mary is vacationing in Salt Lake City, which is near a famous lake known for its high concentration of salt in the water. Mary has a mass of m=55 [kg], and an average body density of $\rho = 950$ [kg m^{-3}]. When bathing in the lake, she notices that 30% of her body is above the water level, while the remaining 70% is below the water level.

(a) What is the density of Salt Lake?

Use the balance between buoyant force and weight:

$\rho \cdot V \cdot g = \rho_{SL} \cdot (0.7 \cdot V) \cdot g$

$\Rightarrow \rho_{SL} = \frac{\rho}{0.7} = 1357$

(b) (Extra points) Is the knowledge of Mary's mass necessary? And why?

No, only the knowledge of the density is sufficient.

USEFUL CONSTANTS:

$G = 6.67 \times 10^{-11}$ [N kg^{-2} m^2]
g=9.8 [m s^{-2}]