FINAL EXAM - PH111
SUMMER 2006
August 4, 2006

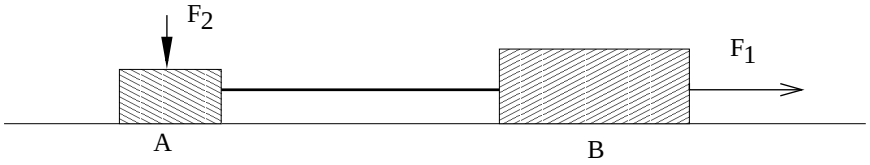
Solve **any 4** of the following 6 problems. Each problem is worth 25 points. Remember to write the full solution to the problem, not just the answer. A bonus question (10 extra points) is also available. Possibly useful informations are reported at the end of the test.

1. A car races along a straight road with a speed of 100 [mi h^{-1}]. A police car spots the speeding vehicle, and starts (at time $t = 0$ [s]) in its pursuit (from rest) when the speeding car is 0.5 [km] ahead of the police car.

(a) Find the time it takes for the police car to overtake the speeding vehicle, if the police car accelerates uniformly with a=1 [m s^{-2}].

(b) The state line is 2.5 mi away from the time the speeding vehicle is spotted by the police car. Will the police car reach the speeding vehicle before it reaches the state line?

2. Two boxes of mass m_A=2 [kg] and m_B=6 [kg] are being pulled by a force of magnitude F_1=40 [N]; another force of magnitude F_2=40 [N] pushes against box A. A massless rope connects the two boxes, the surface has a coefficient of kinetic friction μ_K=0.25.

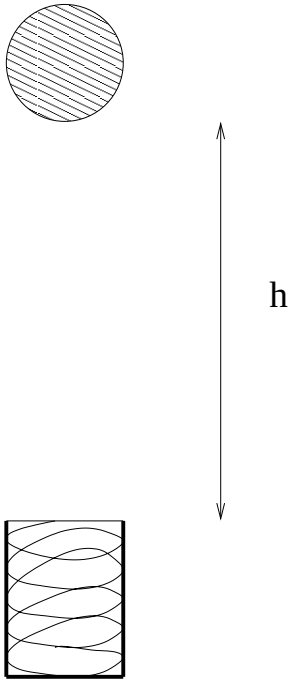


(a) Find the acceleration a of the two boxes;

(b) Find the velocity of the two boxes after they travel a distance of 1 [m];

(c) How much energy has been dissipated into heat during the 1 [m] displacement?

3. A ball of mass m=1 [kg] is released from a height of h=10 [m] above a spring of (unknown) elastic constant k.

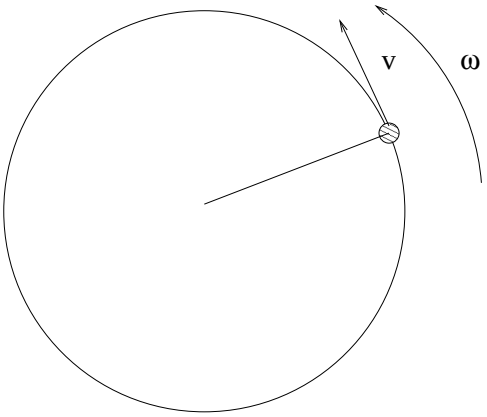


(a) The maximum compression of the spring is found to be x=25 [cm]. Find the elastic constant of the spring.

(b) Find the net force acting on the ball at the moment when the spring is at its maximum compression.

(c) Assume now that there is a constant frictional force of magnitude F_f=20 [N] acting on the ball as it slides down the shaft of the spring. Find the maximum compression of the spring in this case. Use the same value of k you found in part (a).

4. A person (m_p=70 [kg]) sits at the edge of a merry-go-round, which is a uniform disc of radius 5 [m] and mass M=500 [kg]. The merry-go-round has an initial angular speed of 1 [rad/s].

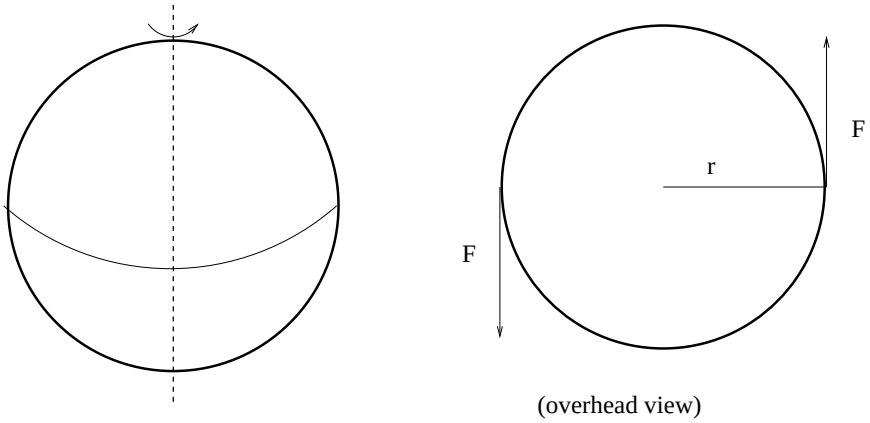


At a given time, the person jumps off the merry-go-round, with a velocity of v=2 [m/s], directed perpendicular to the merry-go-round, as in Figure, and in the same direction as the rotation of the merry-go-round.

(a) Find the angular momentum of the merry-go-round+person *before* the jump, and the angular momentum of the person and of the merry-go-round *after* the jump.

(c) Find the angular velocity of the merry-go-round *after* the jump.

5. A whole metal sphere of radius R=1 [m] and mass m=120 [kg] can rotate about a vertical axis through its center. Two forces of equal magnitude F=30 [N] act on the sphere, in the direction shown in Figure. The two forces remain tangent to the sphere as the sphere begins its motion.



(a) Find the angular acceleration of the sphere;

(b) After how many seconds will the sphere reach an angular velocity of $\omega = 10$ [rad/s]?

6. A star ($M_s = 2 \cdot 10^{30}$ [kg]) is located at a distance of $r = 3 \cdot 10^{20}$ [m] from the center of the Galaxy, and it moves in a circular orbit of $v = 100$ [km/s] around the center of the galaxy.

(a) Find the angular velocity and period of rotation of the star around the galaxy.

(b) Find the mass of the Galaxy (defined as the mass from the center of the Galaxy out to the star's position).

Bonus question Torricelli's experiment measured the atmospheric pressure to be $p_{atm} = 1.01 \cdot 10^5$ [Pa] at sea level (assume ρ_{air}=1 [kg/m^3]). Another experiment is to be performed at a height of 2 [km] from sea level. For this purpose, a hot-air baloon is filled with helium (ρ_{He}=0.8 [kg/m^3]), the baloon has a total mass (including the helium) of $m = 200$ kg. The density of the atmosphere is assumed to be *constant* with altitude.

(a) Given that the baloon rises with a constant acceleration of $a = 1$ [m/s^2], find the volume of the baloon (neglect the presenc of the gondola etc.).

(b) What is the measurement of pressure that will be recorded by the baloon at a height of 2 [km] from sea level?

Possibly useful information:

- moment of inertia of uniform disc around its center: $I = 1/2 \cdot M \cdot R^2$.

- moment of inertia of whole sphere around a vertical axis through its center: $I = 2/5 \cdot M \cdot R^2$.

- value of gravitational constant: $G = 6.67 \cdot 10^{-11}[Nm^2kg^{-2}]$.

- value of gravitational acceleration near the Earth's surface: g=9.8 [m/s^2].

- conversion from miles to meters: 1 [mi] = 1600 [m].

- There are 360 days in 1 year, 24 hours in 1 day, 60 minutes in 1 hour, and 60 seconds in 1 minute.