PH661 – Fall 2008

Statistical methods for physics and astrophysics

SOLUTIONS to Assignment #1

1. Consider 3 baseball players:

Player A was at bat 200 times, hitting .310;

player B was at bat 250 times, hitting .290;

player C was at bat 300 times, hitting .265.

(a) The probability that when either player A, B or C were at bat, the ball was hit safely;

P(hit/A)=0.31; P(hit/B)=0.29; P(hit/C)=0.265;

P(A at bat)≡P(A)=200/750=0.267;

P(B)=250/750=0.333;

P(C)=300/750=0.400;

Use theorem of total probability:

P(hit)=Σ_i *P(hit/i)P(i) = 0.31*0.267+0.29*0.333+0.400*0.265=0.285;*

(b) The probability that, given that a hit was recorded, it came from player A, B or C.

Use Bayes' theorem:

$P(A/hit)=\frac{P(hit/A)\cdot p(A)}{P(hit)}=0.290;$

P(B/hit)=0.339;

P(C/hit)=0.372.

2. Consider two events A and B. Prove the following property of probability, based on the Kolmogorov axioms:

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

First, realize that $A \cup B = (A \cap \bar{B}) \cup (B \cap \bar{A}) \cup (A \cap B)$, *in which the three sets are mutually exclusive.*

$\Rightarrow P(A \cup B) = P(A \cap \bar{B}) + P(B \cap \bar{A}) + P(A \cap B).$

Then, notice that $\forall A, B \subset \Omega$, $A = (A \cap \bar{B})) \cup (A \cap B)$, *since* $B, \bar{B}$ *is a partition of* Ω. *This implies that* $P(A) = P(A \cap B) + P(A \cap \bar{B})$ *due to the fact that the two sets are again mutually exclusive. It thus follows that*

$$P(A \cup B) = P(A) - P(A \cap B) + P(B) - P(A \cap B) + P(A \cap B) = P(A) + P(B) - P(A \cap B)$$