Abstract

It is a long-standing open question whether the Julia set of some rational function is an indecomposable continuum. By definition, a continuum is **indecomposable** iff it is not the union of two of its proper subcontinua. There are several ways of recognizing intrinsically that a continuum $X$ is indecomposable. For instance, $X$ is indecomposable if and only if every proper subcontinuum of $X$ is nowhere dense in $X$. We provide a condition for testing whether the Julia set of a rational function is an indecomposable continuum using data from its complement. In the complex dynamics context, that means we are investigating the (possibly topologically and dynamically complicated) Julia set from the point of view of the (always topologically and dynamically simple) Fatou set.

To state our theorem we need to define some terms. A **generalized crosscut** of a complementary domain $U$ is an open arc $A \subset U$ such that $\overline{A \setminus A} \subset \partial U$. Let $U$ be a plane domain and $A$ a generalized crosscut of $U$. We call each of the two components of $U \setminus A$ a crosscut neighborhood. If $V$ is a crosscut neighborhood determined by generalized crosscut $A$, we call the continuum $S = \partial V \cap \partial U$ a **shadow** of $A$. A sequence $(U_n)_{n=1}^{\infty}$ of (not necessarily distinct) complementary domains of a continuum $X$ satisfies the **double-pass condition** iff, for any sequence of generalized crosscuts $A_n$ of $U_n$, there is a sequence of shadows $(S_n)_{n=1}^{\infty}$ of $(A_n)_{n=1}^{\infty}$ such that $\lim_{n \to \infty} S_n = X$. We prove the following

**Characterization Theorem:** A plane continuum $X$ is indecomposable iff $X$ has a sequence $(U_n)_{n=1}^{\infty}$ of complementary domains satisfying the double-pass condition.

Recently, Clinton Curry has extended the Characterization Theorem (with an appropriately modified definition of generalized surface crosscut) to continua in compact surfaces.

*Co-Authors: Clinton P. Curry (University of Alabama at Birmingham) and E. D. Tymchatyn (University of Saskatchewan)